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# DOMINANCE AND DEPENDENCE IN INPUT–OUTPUT ANALYSIS: THE NONLINEAR (NETWORK) APPROACH

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**Abstract**—In this paper, the network approach is applied to estimate the relative importance of sectors in a macro economy for which estimates of technical coefficients were originally derived using the Analytic Hierarchy Process. The proposed approach suggestes a plausible alternative, since the presence of nonlinear structural relations of an economic system involving intersectoral feedbacks cannot be examined within the framework of the standard input–output model.

## INTRODUCTION

In Ref. [1] Saaty and Vargas show the application of the Analytic Hierarchy Process (AHP) to estimate the input–output coefficients of an economy. A thorough account of the philosophy, general theory and methodology is given by the originator of the AHP [2].

To estimate the input–output coefficients for a given economy, the AHP is applied in a two-stage procedure [1]:

(a) determine the relative impact of the sectors, based on the *a priori* value of the sectors, on the economy;
(b) determine the interdependence among sectors, where each sector is valued (ratio-scaled) to account for its contribution to the remaining sectors of the economy.

The results of these two steps are then synthesized thereby obtaining the input–output matrix of the estimates of technical coefficients. The application of the AHP procedure produces a table of technical coefficients which can then be used in conjunction with the standard input–output model,

$$X = (I - A)^{-1} Y,$$

to obtain estimates of each sector's output ($X$) based on the exogenously determined values of the demand ($Y$) sectors of a given economy.

## THE NETWORK APPROACH

To illustrate the utility of this approach in the context of input–output analysis, we begin by specifying a *network* of intersectoral relations for a typical economy previously characterized [1] as shown in Fig. 1.

The network structure shown in Fig. 1 consists of three "components": (C1) Agriculture (AGR); (C2) Transportation and Distribution (TD); and (C3) a Cluster C which in turn consists of four "elements"—(c1) Public Utilities (PU), (c2) Mining and Minerals (MM), (c3) Construction (CONS) and (c4) Services (SER).

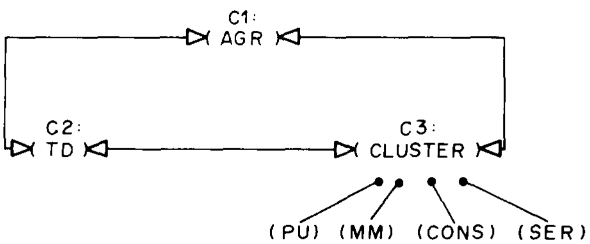
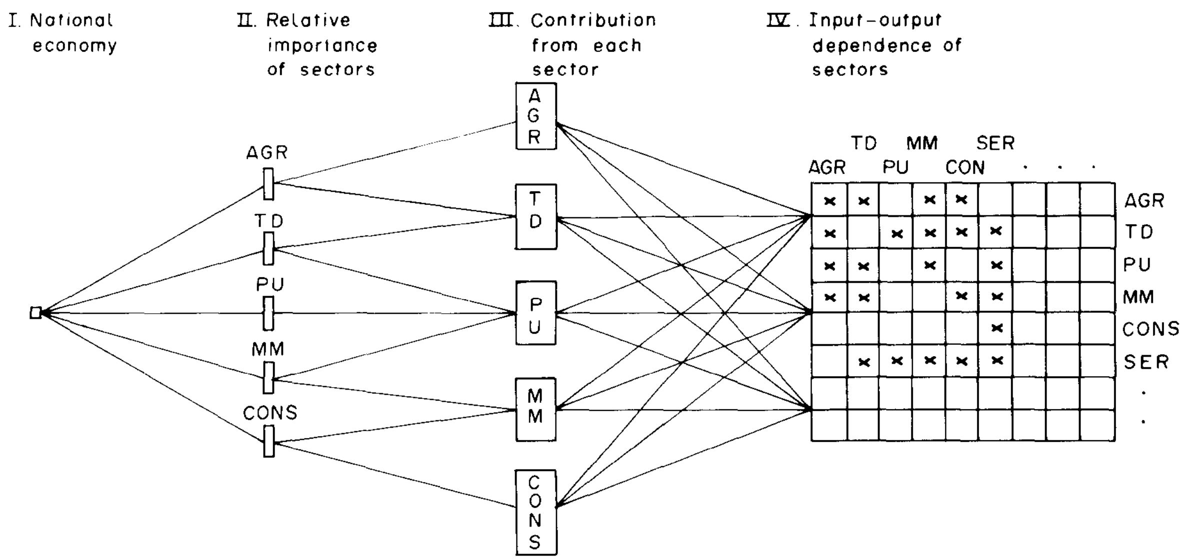



Fig. 1. The network of interactions among sectors.

The intersectoral impact(s) are indicated by the directions of the arrows for this illustrative network. This network can be used at Stage II of Fig. 2, showing explicitly the pattern of interactions among sectors of the economy with data given in Ref. [1], while measuring the relative dominance of sectors. Figure 2 expresses schematically the conceptual framework of the AHP when used as a process of determining the input–output relations of a given economy.

In general, interactions in a network can be represented by the matrix $W$ [3]:

$$W = \begin{array}{cc} & \begin{array}{ccc} C_1 & \cdots & C_n \\ c_{11}c_{12}\cdots c_{1n1} & & c_{N1}c_{N2}\cdots c_{Nn_N}\end{array} \\ \begin{array}{cc} C_1 & \begin{array}{c} c_{11}\\ c_{12}\\ \vdots \\ c_{N1}\end{array} \\ \vdots & \\ C_N & \begin{array}{c} c_{N1}\\ c_{N2}\\ \vdots \\ c_{Nn_N}\end{array}\end{array} & \begin{bmatrix} W_{11} & \cdots & W_{1n} \\ \vdots & & \vdots \\ W_{N1} & \cdots & W_{NN}\end{bmatrix}\end{array}$$




COMBINED METHODS

1. Eigenvalue method (EM):

$Aw = nw$

$A: a_{ij} = 1/a_{ji}; a_{ii} = 1$

$\sum_{i=1}^{n} \lambda_i = \mathrm{tr}(A) = n$

$\lambda_{\max} \neq \lambda_i = 0$

$Aw = \lambda_{\max} \cdot w$

$a_{ik} = a_{ij} \cdot a_{jk}$: condition of transitivity

$[n(n-1)]/2$: number of judgments

$[\lambda_{\max} - n]/[n-1]$: test of consistency

2. Networks:

$Ww = w$

$w(k) = \lim_{k\to\infty} W^k$: limiting impact priority (LIP)

3. Input–output:

$X = (I - A)^{-1}Y$

Fig. 2. The structure of interactions among sectors of a national economy.

where the $(i,j)$ block is given by

$$W_{ij} = \begin{bmatrix} w_{i1}^{(j1)} & w_{i1}^{(j2)} & \cdots & w_{i1}^{(jn_j)} \\ w_{i2}^{(j1)} & w_{i2}^{(j2)} & \cdots & w_{i2}^{(jn_j)} \\ \vdots & \vdots & \cdots & \vdots \\ w_{in_i}^{(j1)} & w_{in_i}^{(j2)} & \cdots & w_{in_i}^{(jn_j)} \end{bmatrix}.$$

Each column of $W_{ij}$ represents the relative impact of the elements in the $i$th component on each of the elements in the $j$th component.

Following the process outlined in Ref. [1] we construct the following matrices to determine the interrelations of the sectors and their relative impacts on the Agriculture (AGR), Transportation and Distribution (TD) and the "rest of the sectors" Cluster C in the economy. Using the data given in Ref. [1], for the Agriculture and Transportation and Distribution sectors we have:

$$\begin{bmatrix} \text{AGR} & \text{PU} & \text{MM} & \text{CONS} & \text{SER} & \text{Eigenvector} & \text{TD} & \text{PU} & \text{MM} & \text{CONS} & \text{SER} & \text{Eigenvector} \\ \text{PU} & 1 & 1/9 & 3 & 2 & 0.1552 & \text{PU} & 1 & 1 & 2 & 9 & 0.3920 \\ \text{MM} & 9 & 1 & 4 & 9 & 0.6708 & \text{MM} & 1 & 1 & 1 & 7 & 0.3077 \\ \text{CONS} & 1/3 & 1/4 & 1 & 4 & 0.1212 & \text{CONS} & 1/2 & 1 & 1 & 7 & 0.2593 \\ \text{SER} & 1/2 & 1/9 & 1/4 & 1 & 0.0052 & \text{SER} & 1/9 & 1/7 & 1/7 & 1 & 0.0040 \end{bmatrix},$$

$$\begin{bmatrix} \text{C: PU} & \text{PU} & \text{CONS} & \text{SER} & \text{Eigenvector} & \text{C: MM} & \text{PU} & \text{CONS} & \text{SER} & \text{Eigenvector} \\ \text{MM} & 1 & 9 & 5 & 0.7534 & \text{PU} & 1 & 1/2 & 1/3 & 0.1692 \\ \text{CONS} & 1/9 & 1 & 5 & 0.1741 & \text{CONS} & 2 & 1 & 1 & 0.3873 \\ \text{SER} & 1/5 & 1/5 & 1 & 0.9624 & \text{SER} & 3 & 1 & 1 & 0.4435 \end{bmatrix}$$

$$\begin{bmatrix} \text{C: CONS} & \text{PU} & \text{MM} & \text{SER} & & \text{C: SER} & \text{PU} & \text{MM} & \text{CONS} & \\ \text{PU} & 1 & 2 & 2 & 0.5 & \text{PU} & 1 & 1/2 & 3 & 0.3090 \\ \text{MM} & 1/2 & 1 & 1 & 0.25 & \text{MM} & 2 & 1 & 5 & 0.3816 \\ \text{SER} & 1/2 & 1 & 1 & 0.25 & \text{CONS} & 1/3 & 1/5 & 1 & 0.1094 \end{bmatrix}.$$

The supermatrix $W$ containing the eigenvectors measuring the interactions of sectors obtained from previous matrices is given by

$$W = \begin{bmatrix} & [0.311] & [0.493] & \multicolumn{4}{c}{\text{Cluster C: rest of the economy } [0.195]} \\ & \text{AGR} & \text{TD} & \text{PU} & \text{MM} & \text{CONS} & \text{SER} \\ \text{AGR} & 1 & 1 & 1 & 1 & 1 & 1 \\ \text{TD} & 1 & 1 & 1 & 1 & 1 & 1 \\ \text{PU} & 0.1552 & 0.3920 & 0 & 0.1692 & 0.5 & 0.3090 \\ \text{MM} & 0.6708 & 0.3077 & 0.7534 & 0 & 0.25 & 0.5816 \\ \text{CONS} & 0.1212 & 0.2593 & 0.1741 & 0.3873 & 0 & 0.1094 \\ \text{SER} & 0.0052 & 0.0040 & 0.0724 & 0.4435 & 0.25 & 0 \end{bmatrix}.$$

The numbers in square brackets above indicate the weights of the three components taken directly

Table 1. The pairwise comparison matrix from Ref. [1]

| Contribution to the economy | AGR | TD | Cluster C | Component weight |
|---|---|---|---|---|
| AGR | 1 | 1/2 | 2 | [0.312]: C1 |
| TD | 2 | 1 | 2 | [0.493]: C2 |
| Cluster | 1/2 | 1/2 | 1 | [0.195]: C3 |

from the eigenvector results of the pairwise comparison matrix already given in Ref. [1] (see Table 1).

To transform $W$ into a stochastic matrix we multiply the first row by 0.312, the second by 0.493 and the rest of the rows by 0.195:

$$\begin{array}{c} \\ \text{AGR} \\ \text{TD} \\ \text{PU} \\ \text{MM} \\ \text{CON} \\ \text{SER} \end{array} \begin{bmatrix} \text{AGR} & \text{TD} & \text{PU} & \text{MM} & \text{CON} & \text{SER} \\ 0.3120 & 0.3120 & 0.312 & 0.3120 & 0.3120 & 0.3120 \\ 0.4930 & 0.4930 & 0.493 & 0.4932 & 0.4931 & 0.4940 \\ 0.0302 & 0.0764 & 0 & 0.0329 & 0.0975 & 0.0602 \\ 0.1308 & 0.0687 & 0.1469 & 0 & 0.0487 & 0.1134 \\ 0.0236 & 0.0505 & 0.0339 & 0.0755 & 0 & 0.0213 \\ 0.0014 & 0.0007 & 0.0142 & 0.0864 & 0.0487 & 0 \end{bmatrix}.$$

Raising the resulting matrix to powers we obtain the following matrix of the overall (economic) system weights:

$$\begin{array}{c} \\ \text{AGR} \\ \text{TD} \\ \text{PU} \\ \text{MM} \\ \text{CON} \\ \text{SER} \end{array} \begin{bmatrix} \text{AGR} & \text{TD} & \text{PU} & \text{MM} & \text{CONS} & \text{SER} \\ 0.3143 & 0.3143 & 0.3143 & 0.3142 & 0.3142 & 0.3143 \\ 0.4930 & 0.4930 & 0.4930 & 0.4930 & 0.4930 & 0.4930 \\ 0.0540 & 0.0542 & 0.0560 & 0.0596 & 0.0516 & 0.0529 \\ 0.0814 & 0.0881 & 0.0777 & 0.0928 & 0.0943 & 0.0843 \\ 0.0434 & 0.0400 & 0.0436 & 0.0352 & 0.0402 & 0.0428 \\ 0.0136 & 0.0102 & 0.0151 & 0.0049 & 0.0063 & 0.0124 \end{bmatrix}$$

Now taking any one column of this matrix (as they are approximately identical) and comparing with the results obtained by Saaty and Vargas [1] on the relative importance of sectors:

| | | AGR | TD | PU | MM | CONS | SER |
|---|---|---|---|---|---|---|---|
| AHP | [1] | 0.3108 | 0.4934 | 0.0248 | 0.0546 | 0.0546 | 0.0608 |
| LIP | [any column] of $W$ | 0.3143 | 0.4930 | 0.0560 | 0.0777 | 0.0436 | 0.0151 |

Note that the sectors' total relative index of importance obtained by the two methods are close. This provides evidence, and corroborates the network structure initially assumed for the typical economy.

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